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ANALYTICAL INVESTIGATION OF MULTINOZZLE PLUME FLOW FIELDS - FLO--ETC(U)
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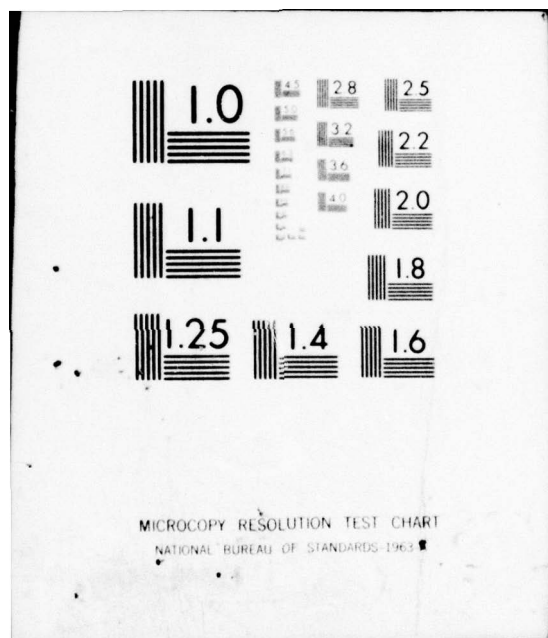
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INTERIM SCIENTIFIC REPORT

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ANALYTICAL INVESTIGATION OF MULTINOZZLE
PLUME FLOW FIELDS - FLOW STRUCTURE AND COMPUTATION

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Detailed knowledge of jet and rocket engine exhaust flow fields is sought in a wide variety of military and civilian programs. The prediction of infrared signature, radar cross section, electromagnetic wave alternation, and production and dispersion of noxious pollutants requires the ability to predict the spatial distribution of all thermodynamic and flow quantities. All present plume models are based on single nozzle flow fields in which an "equivalent" engine is defined which has the combined thrust of all the individual engines. A great majority of vehicles of interest have either multiple engines or multiple exhaust nozzles leading to both near and far field flow properties not properly portrayed by the single nozzle concept. The nozzles generally operate at underexpanded conditions so that the individual exhaust plumes impinge. Highly complex flow fields arise containing a number of three dimensional shock surfaces and shock wave plume boundary interactions. The present research program is directed toward the accurate prediction of these plumes.

Considerable progress has been achieved in the multiple nozzle plume flow field research effort in both the qualitative understanding of the phenomena and the development of a three dimensional numerical computation technique for the quantitative prediction of these flow fields. The achievements in these areas are summarized in the two appendices of this report which are copies of abstracts submitted for presentation at upcoming AIAA meetings. Appendix A is a copy of the abstract "Multinozzle Plume Flow Fields - Structure and Numerical Calculation" sent to the 10th AIAA Fluids and Plasma Conference discusses the flow field structure and will present numerical computations for the impingement of two uniform (rectangular cross section) plumes. These numerical calculations are expected shortly since an isentropic computer code is already operational and the shock and pressure boundary routines are coded and require only final debugging. The shock structure observed on all available photographs of twin plume impingement can now be explained as a result of recognition of the existence of an additional recompression shock system. The interaction of this shock system with the barrel and impingement shocks explain all the unusual features of the twin plume flow field. The second abstract, Appendix B, "Three Dimensional Boundary Conditions in Supersonic Flow" was submitted to the 3rd AIAA Computational Fluids Meeting. This effort, which is a necessary step in the development of the

required computational effort details a major development in the application of three dimensional boundary conditions in supersonic flow. This paper also describes a new method for exact (MOC boundary point calculation.

Considerable progress has been made to date, as documented in the appendices. However, this research effort encountered conceptual problems associated with two features of the plume impingement problem which required more time than was initially expected. The first of these is associated with the flow details at the point where the impingement shock intersects the plume boundary. There results a centered expansion exactly at this point to rebalance the pressure at the plume interface. In some cases, this process takes place under transonic flow conditions where the expansion can immediately reflect off the sonic line and thus makes the flow angularity at the plume boundary indeterminant by supersonic flow calculations methods. References were found that deal with centered transonic expansions. It turns out that the flow is initially a Prandlt Meyer expansion so that at least on theoretical grounds there is no obstacle.

The other problem has to do with the imposition of initial conditions. The initial station is taken at a point where the impingement shock detaches from the symmetry plane and climbs the undisturbed plume boundary. The initial zone of influence of the detachment point is a conical flow which is essentially governed by elliptic equations. Thus the distribution of properties in this zone cannot be determined a priori and can only be estimated. In the past, for flow over a cone, previous methods, estimated the initial conditions and marched far downstream to a point where the computed flow field was conical. These flow properties could then be reinterpreted in conical coordinates and used as initial conditions at the tip of the cone. This can be done in the case of a cone because the geometry and boundary conditions are the same in conical coordinates to infinity. In the case of the impingement shock detachment problem, this convenient geometrical situation is not available. For this reason the impingement of two uniform circular jets which was originally contemplated at this stage of the work, was replaced with the impingement of two uniform rectangular jets. In this latter case, we can achieve the conical flow situations and perhaps adapt this solution to the more complex problem.

In summary, considerable progress has been achieved in both the understanding of the structure of multinozzle flow fields and the development of numerical procedures for their calculation. A three dimensional isentropic flow computer code is operational. The computation of the flow field associated with the impingement of two uniform (rectangular cross section) plumes is coded and results are expected very shortly. Two abstracts have been submitted to AIAA meetings reporting on this research program.

Appendix A

MULTINOZZLE PLUME FLOW FIELDS - STRUCTURE AND NUMERICAL CALCULATION *

S. Rudman**

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Detailed knowledge of jet and rocket engine exhaust flow fields is sought in a wide variety of military and civilian programs. The prediction of infrared signature, radar crosssection, electromagnetic wave alternation, and production and dispersion of noxious pollutants requires the ability to predict the spatial distribution of all thermodynamic and flow quantities. All present plume models are based on single nozzle flow fields in which an "equivalent" engine is defined which has the combined thrust of all the individual engines. A great majority of vehicles of interest have either multiple engines or multiple exhaust nozzles leading to both near and far field flow properties not properly portrayed by the single nozzle concept. The nozzles generally operate at underexpanded conditions so that the individual exhaust plumes impinge. Highly complex flow fields arise containing a number of three dimensional shock surfaces and shock wave plume boundary interactions. Despite the interest in the multinozzle flows, there exists only a small experimental data base and only minor progress has been made along analytical and theoretical lines. This paper describes a current research effort which has made significant steps toward the understanding and prediction of these flows. The flow structure associated with the impingement of two under-expanded rocket plumes is explained and three distinct possibilities appear. The prediction of these complex flow fields - the goal of this effort - requires the development of new computational procedures. This method, a development of previous fitted shock techniques, (eg Ref. 1-4) is based on finite difference methods which allow for a large number of flow discontinuities. Discrete shock waves, pressure boundaries and more complex singularities are permitted to float

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** Senior Research Scientist

between the computational mesh and are computed employing a new three dimensional boundary point calculation procedure (Ref. 5). Numerical results for the impingement of two uniform rectangular jets will be presented. This flow field demonstrates a majority of the features of underexpanded plume impingement flow fields and of the numerical calculation procedures. No part of this analysis or numerical results has been published previously.

The structure of the multinozzle plume flow field contains an extremely complex pattern of shock waves which are governed by three dimensional considerations. It has become clear during the present research effort that there are three major shock systems in the multinozzle plume flow field. In addition to the barrel shock (B shock) and impingement shock (I), there is a recompression shock (R) which spreads laterally at first to relieve the high pressure region under the I shock. The barrel shock forms in the single nozzle plume because the expansion system at the nozzle lip is axisymmetric giving rise to conical wave fronts. The spreading bicharacteristics on these wave fronts produces additional expansion in the flow which upon interaction at the constant pressure plume interface result in reflected compression waves. These eventually focus to start the barrel shock system (B1) (see Fig. 1). Because of the axisymmetric nature of the flow the wave strength of the B1 shock increases as it approaches the axis of the plume and results in a Mach disc and reflected shock system (B2). The flow behind the Mach disc is subsonic so that the location of the disc depends on expansion and mixing processes downstream of it. This is in distinction to the remainder of the flow which is supersonic and where there is no upstream influence. This inviscid flow pattern is well understood and several computer codes are available (in varying degrees of approximation) to predict it (eg Ref. 6).

The flow pattern of the multiple nozzle plume has two additional shock wave systems. The shock structure of a uniform twin jet impingement, Fig. 2, is quite informative in the nature of the impingement shock (I) and recompression shock (R) systems. In the side view the I shock appears basically as expected from a two dimensional pattern. A complex process takes place at the intersection of the plume boundary and the I shock. Based on work by Hunt and co-workers (Ref. 7, 8, and 9) the discontinuous boundary pattern sketched in Fig. 2 is expected. These references deal with normal impingement of uniform jets; however, the interaction of the I shock and the plume boundary is locally equivalent to that case. An expansion emanates from the plume boundary at the point of impingement,

to cancel the pressure rise due to the I shock wave (Station 1). At Station 2, a new feature develops in the flow — expansion wave fronts stretching in three dimensions interact with the constant pressure boundary giving rise to inward moving compression wave surfaces that coalesce to form a recompression (R) shock system. This coalescence is completely analogous to the formation of the barrel shock (B) system in the axisymmetric case. Subsequently, (Station 3-5) the R shock system shrinks in size and grows in strength as it approaches the plume center.

The impingement of two underexpanded plumes, in general, contains the three shock systems discussed above which are further distorted by the spatial non-uniform flow properties. Many shock configurations are possible depending on the relative strengths of the three systems and the order in which they intersect. Three observed (Ref. 10) configurations will be described below. Each schematic is followed by a glow photograph (Ref. 10) (these copies are of low quality and are not clear in some cases - in the full paper high quality reproduction will be used). In Type I (Fig. 3) the first Mach cell is only slightly distorted by the I shock (see top view) and the next major shock pattern which occurs in the central region of the flow involves the reflection of the R shock from the horizontal symmetry plane. The reflected B and R shocks interact as seen in the top view. In Type II (Fig. 4) the I shock appears stronger than in Type I situation and in the top view appears to cross the B shock without the formation of a Mach disc. The transmitted B shock and the R intersect in such a way as to create a Mach disc in the flow between the two nozzles. The appearance of this Mach disc seems to be due to the confluence of the transmitted B and R shocks in the same region of the plume. The reproduction of the glow photo (Fig. 4b) is rather poor but the central Mach disc is evident in both views. In Type III (Fig. 5) the transmitted B shock reflects off the vertical symmetry plane (top view point A) while the R shock (side view) is still far from the axis (of the two plumes). Subsequently, downstream, the R shock intersects the reflected B shock system and we see an irregular shaped leading edge for this intersection at Station 2, which is detailed further in the cross section shown for Station 2. The reflected B shock and the two R shocks are not planar and give rise to a twin peaked cross sectional pattern.

The computational procedure for the prediction of the multinozzle plume flow field is based on a forward marching finite difference technique. A standard

second order algorithm is employed at all "interior" points (Figure 6) in conjunction with fitted shocks and discrete boundaries. The floating shock approach devised by Moretti (Ref. 11) and employed by Salas (Ref. 12) is extended here to steady three dimensional flow. A key point is the proper orientation of the characteristic plane in the calculation of boundary and shock points. It can be shown (Ref. 5) that at any point in a three dimensional flow there is a unique plane (the osculating plane) containing the main bicharacteristics which determine the flow at that point. Employing a simplified method (described in detail in Ref. 5) the characteristic relationship along the main bicharacteristic is determined at the unknown boundary point. As an example, Figure 7 shows a sketch describing the geometry for an imbedded shock point calculation. The normal to the shock at this point is iterated until it is consistent with the characteristic equation. The method does not require mapping the discontinuities to boundaries of the computational mesh and thus provides great flexibility. The movement and properties of discontinuity surfaces are treated as they propagate through the finite difference cells. Thus the cell which contains the I shock impingement on the undisturbed plume boundary (shaded cell in Fig. 6) is handled within the general framework of the method. The calculation of this cell combines a shock point calculation, a pressure boundary point calculation and a centered expansion fan at the impingement point. An isentropic three dimensional flow computer program is operational and has been employed for core flow calculations issuing from rectangular nozzles. Calculations for the impingement of two rectangular plumes including all the details (shocks, shock/boundary interaction/pressure boundary) will be presented in the full paper.

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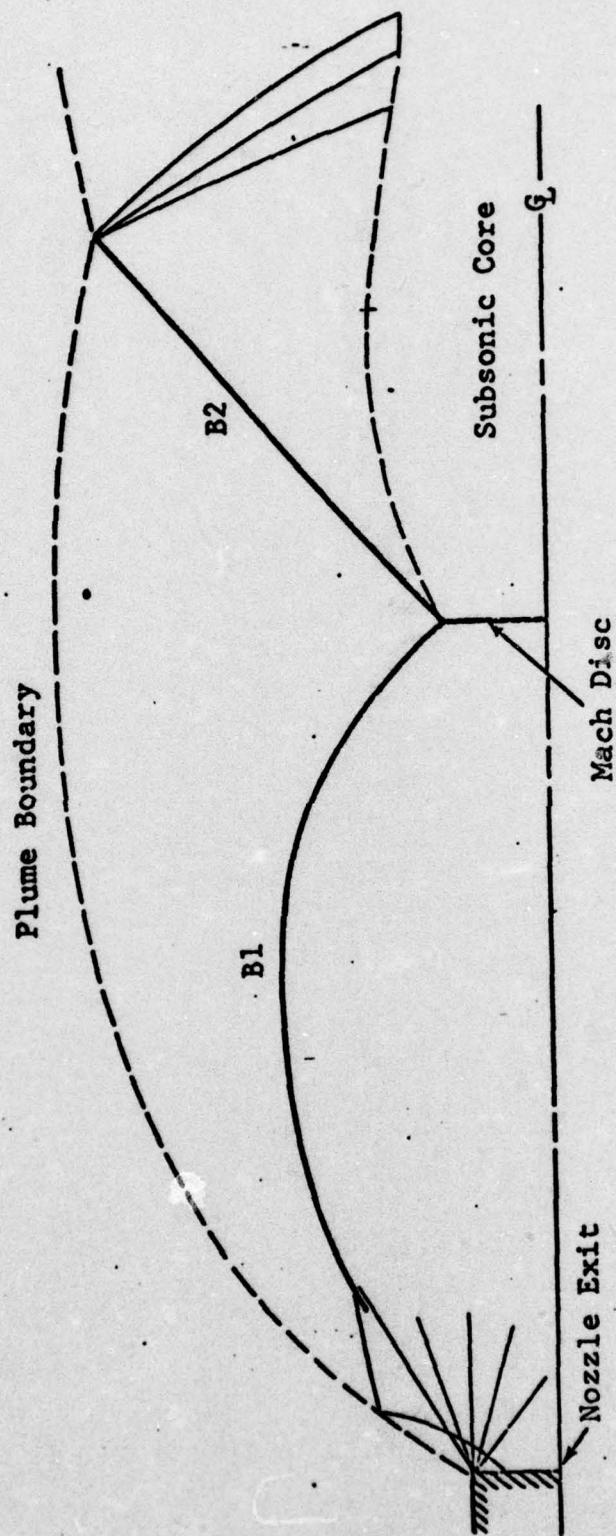
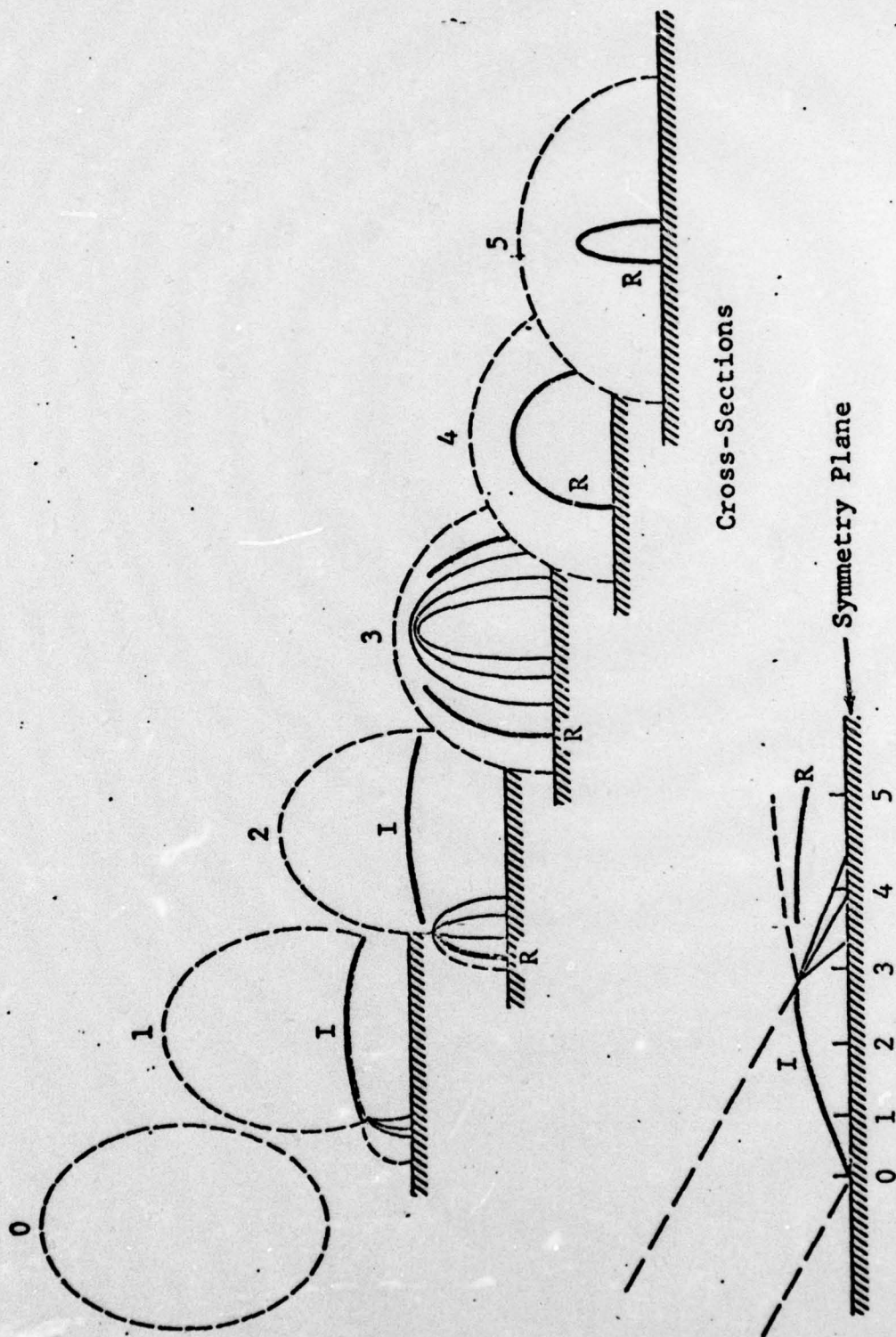
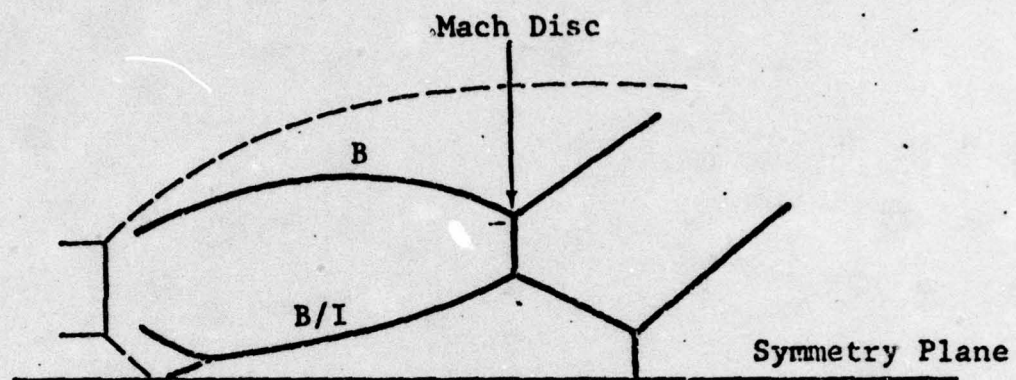


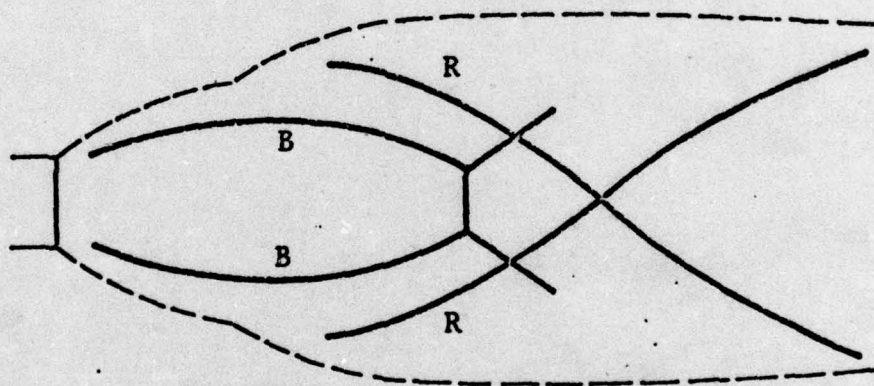
Fig. 1 Single Nozzle Plume Flow Field



Side View
 Fig. 2 Impingement of Two Uniform Plumes



Top View

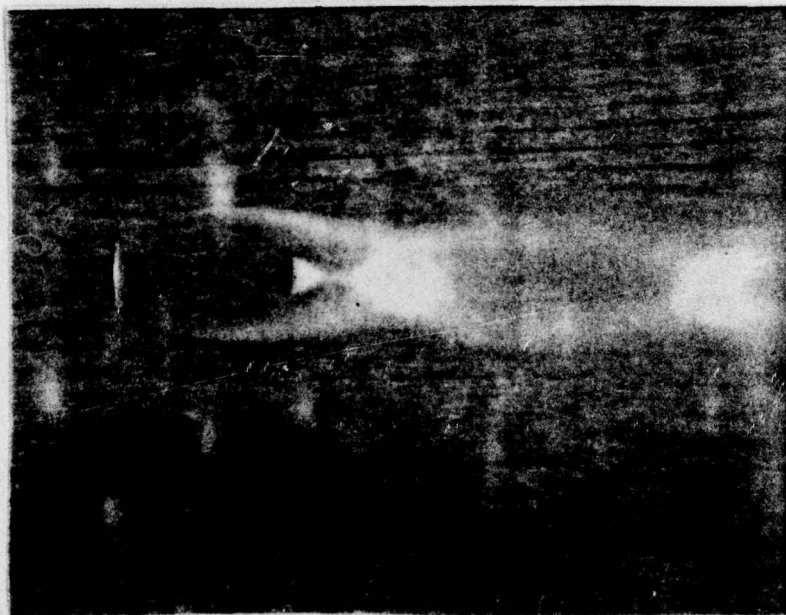


Side View

Fig. 3 Flow Pattern Underexpanded Twin Plumes, Type I
(a) Schematic



Top View



Side View

Fig. 3 Flow Pattern of Underexpanded Twin Plumes, Type I
(b) Glow Photographs

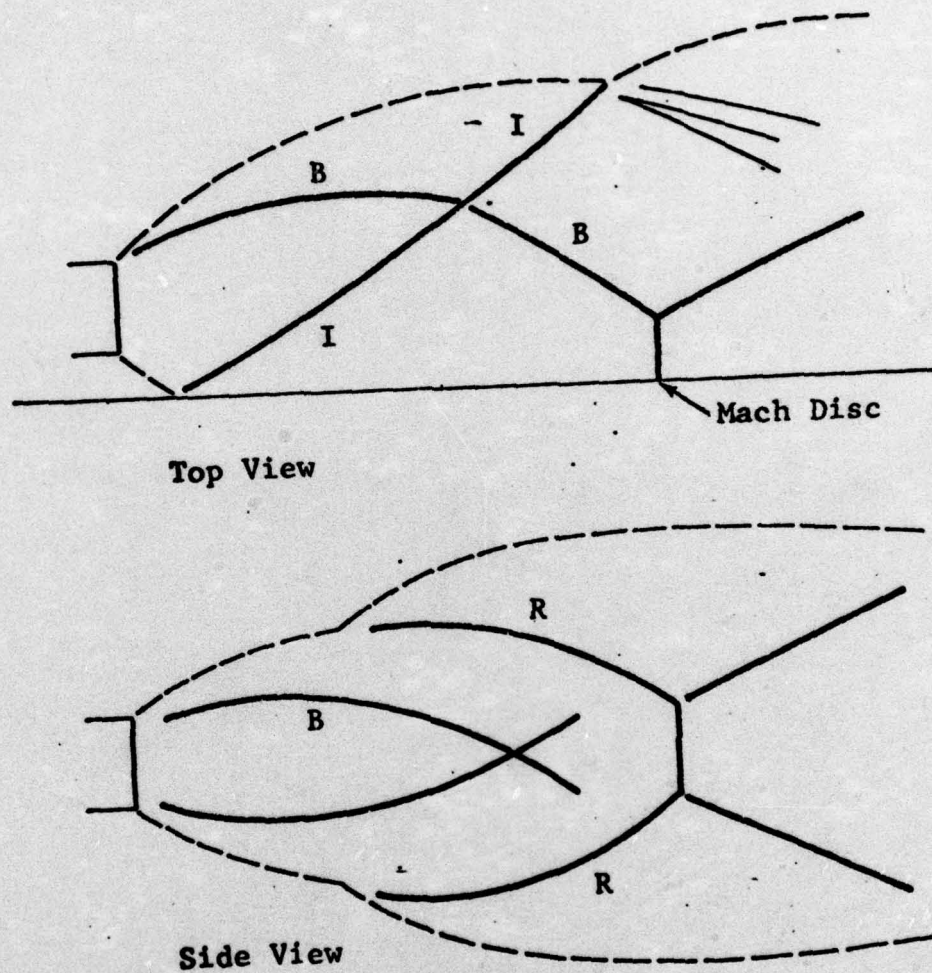
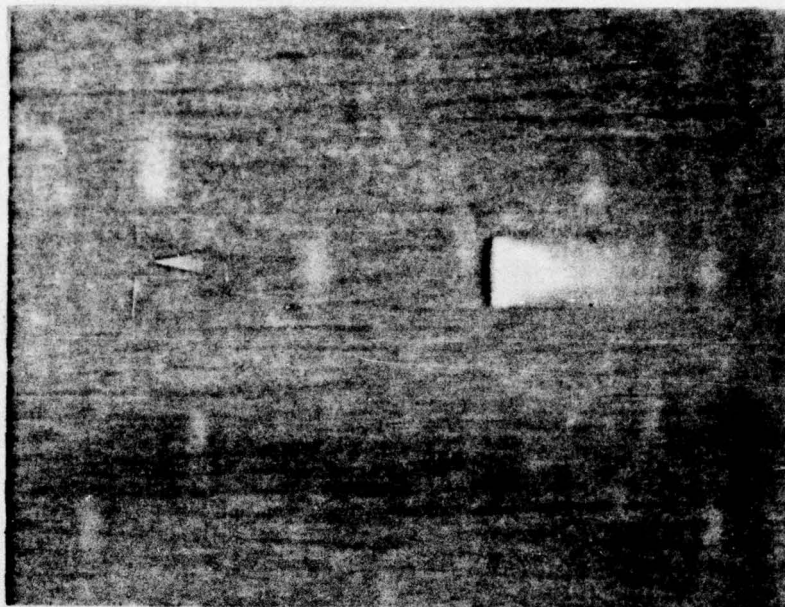


Fig. 4 Flow Pattern of Underexpanded Twin Plumes, Type II
(a) Schematic



Top View



Side View

Fig. 4. Flow Pattern of Underexpanded Twin Plumes, Type II
(b) Glow Photograph

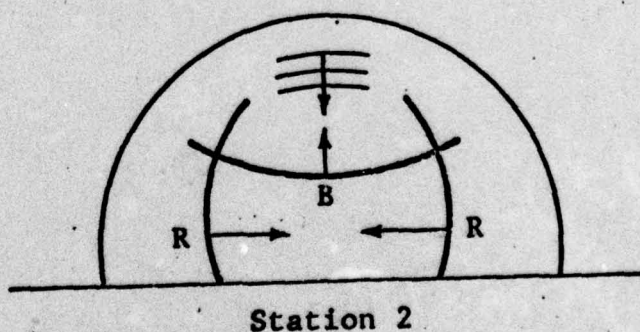
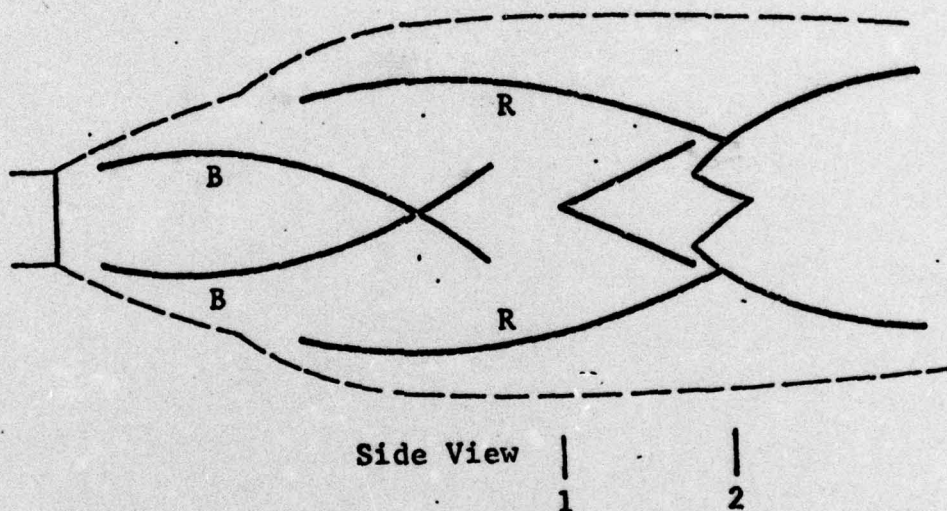
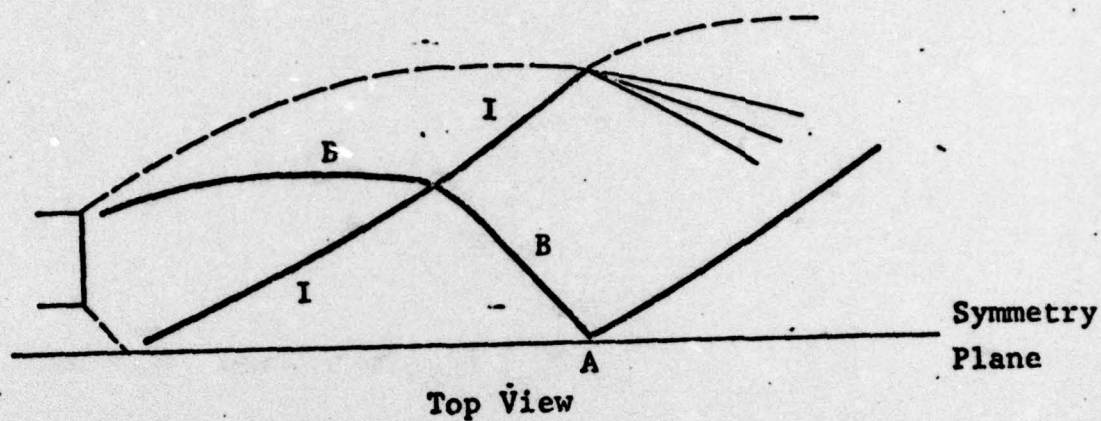
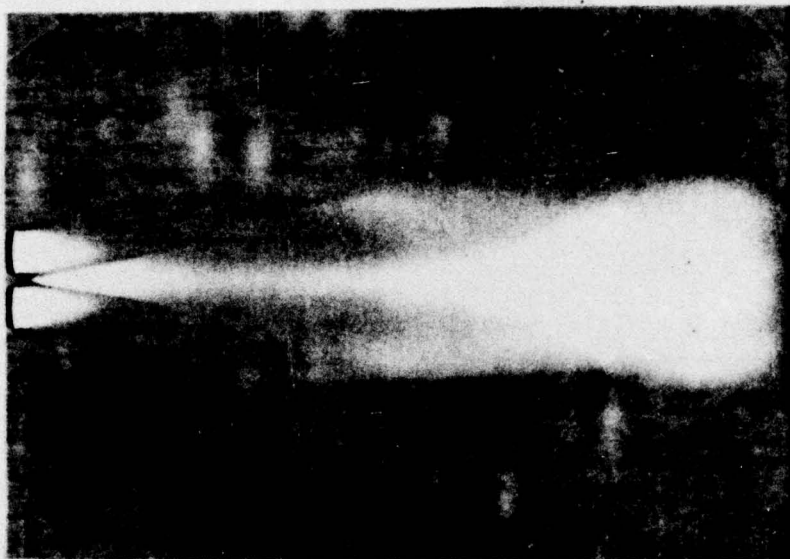
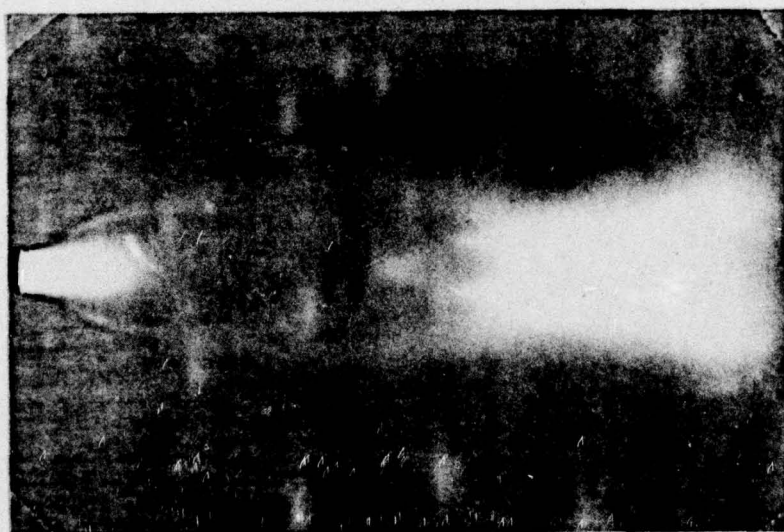


Fig. 5 Flow Pattern Underexpanded Twin Plumes, Type III
(a) Schematic



Top View



Side View

Fig. 5 Flow Pattern of Underexpanded Twin Plumes, Type III
(b) Glow Photographs

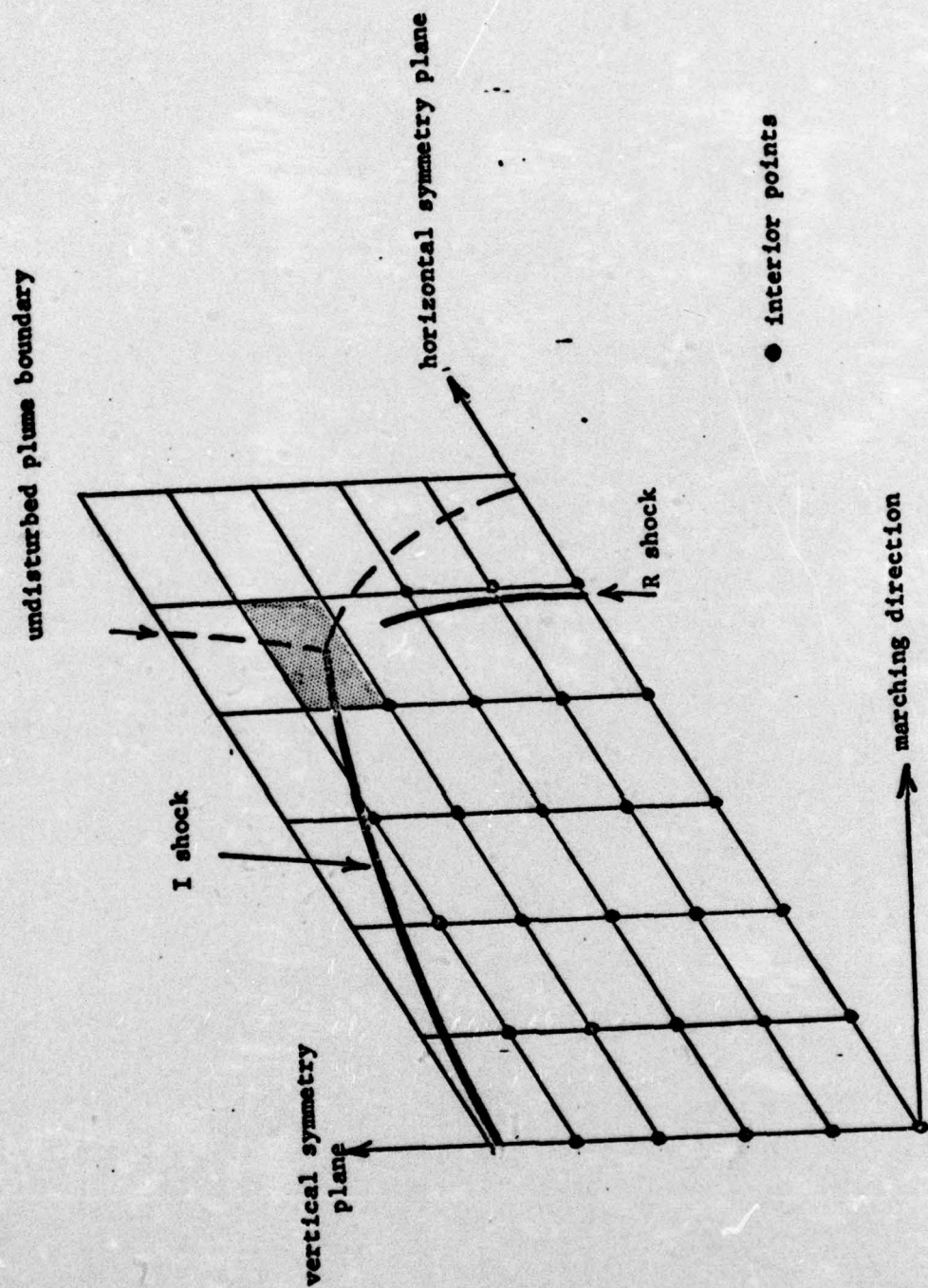


Fig. 6 Computational Mesh Showing Discontinuity Surfaces

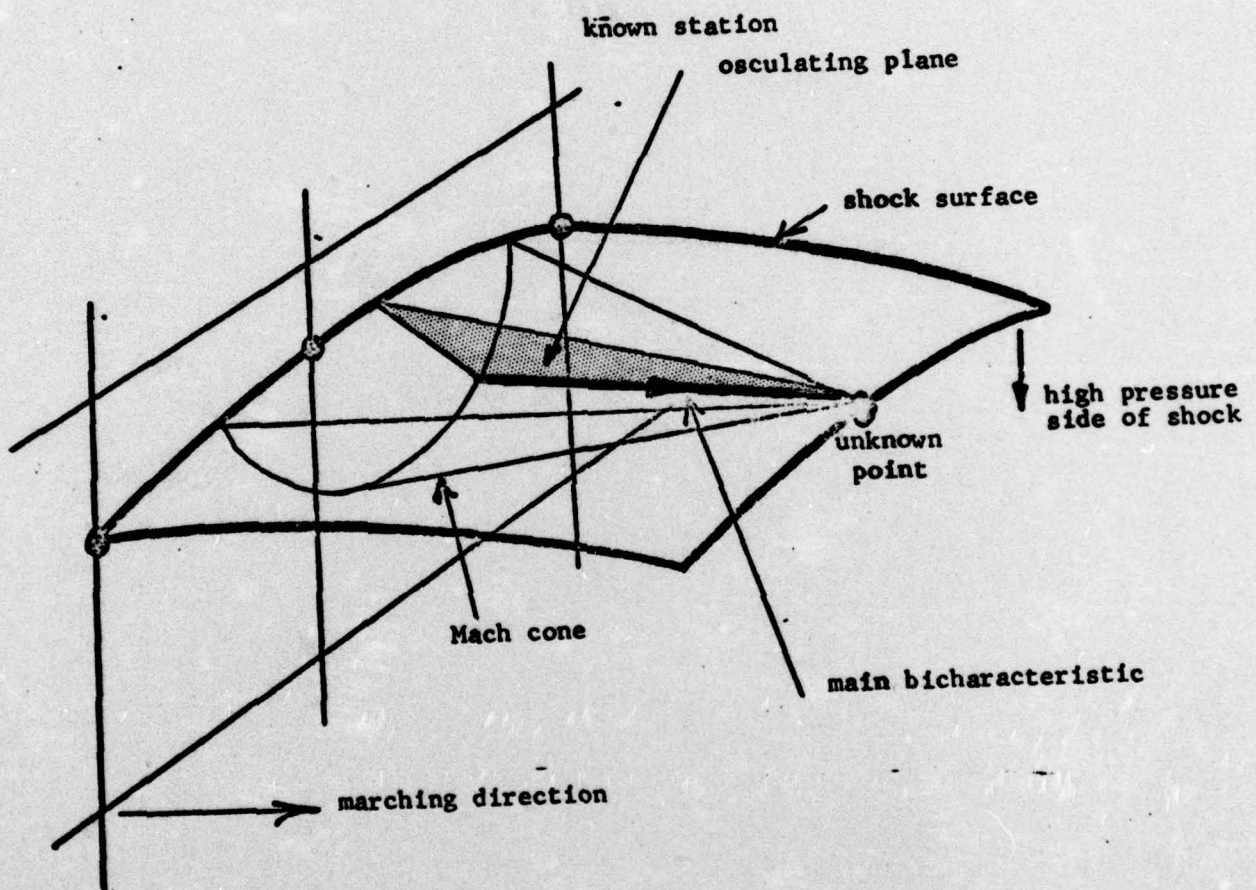


Fig. 7 Schematic Showing Geometry for Shock Point Calculation

THREE DIMENSIONAL BOUNDARY CONDITIONS IN SUPERSONIC FLOW*

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Future success in solving and understanding complex three dimensional supersonic flow problems by numerical techniques will require increased understanding of such flows, and in particular answers to questions concerning the correct imposition of boundary conditions are required. A variety of three dimensional boundary point algorithms have been developed (Ref. 1-5) which employ a two dimensional plane (reference plane) in the vicinity of the boundary in which the calculation is performed. The orientation of this plane relative to the boundary surface has been chosen, in the past, based on intuitive and ad hoc reasoning.

This paper shows that the choice of the proper plane is not arbitrary, rather, there are theoretical grounds which single out a unique plane (Ref. 6). The intersection of this plane (b plane) and the fore Mach cone (at any point in the flow) are the two bicharacteristics which carry the flow disturbances. Orienting the reference plane (in any of the aforementioned schemes) to contain these main (or distinguished) bicharacteristics removes the ambiguity from three dimensional boundary point calculations and should increase their reliability. In addition, a new characteristic type boundary point algorithm for the computation of fitted bow and imbedded shocks, solid surfaces, pressure surfaces and contact discontinuities has been developed. This simplified method circumvents the need for complex interpolation and detailed tracing of characteristics and easily incorporates the b plane concept. The theory and numerical technique are demonstrated on a series of problems including flow inside a rectangular duct, impingement of two supersonic jets and axisymmetric plumes. All material presented in this paper has not appeared previously in any published work.

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** Senior Research Scientists

Supersonic flow computations are amenable to simple forward marching techniques as the flow is governed by hyperbolic equations. In a very real sense hyperbolic differential equations can be viewed as the means by which the boundary and initial conditions are propagated throughout the remainder of the flow and thus yield the solution. This view point is made especially obvious in the solution of a single homogeneous first order partial differential equation. It therefore follows that the numerical solutions which are attained are only as valid and accurate as the boundary point calculation (c.f. Ref. 7). In the simplest approach, where solid boundaries are coordinate surfaces reflection and symmetry conditions can be employed (i.e. flat plate or cone); in the case of pressure boundaries and shock waves (since the disturbance caused by the body is bounded by the outer shock wave) the free stream condition is imposed as the outer boundary condition. A strong compression zone, occupying several mesh points, which portray the bow shock wave is computed (captured) as part of the solution. For complex geometries and especially those where fine flow definition and computational efficiency is paramount more refined methods are required. These methods involve fitting discrete R-H shocks, contact discontinuities and curved pressure and solid boundaries. Locally a method of characteristics calculation is employed to determine the boundary properties.

In order to enforce the proper conditions in three dimensional flow the appropriate characteristic must be determined and traced to the boundary (or shock). In the past there has been considerable ambiguity about the proper choice because of the infinite number of bicharacteristics available. In general at any point in a three dimensional flow it can be shown that there is a unique plane (b plane) normal to the binormal direction of the local streamline in which the flow is locally of axisymmetric character (two space dimensions). This plane has the following unique properties: the pressure gradient normal to this plane is identically zero, the streamline remains in this plane to second order in the marching direction and the bicharacteristics in this plane are the only two which carry the flow disturbances. It is therefore possible (and correct) that the characteristic plane (b plane) could be parallel to a boundary surface or at any oblique angle depending on the local pressure gradient. The b vector direction (normal to the b plane) can be defined as $\nabla p \times \vec{q}$ and as such is determined as part of the solution.

In the past the complexity of imbedding a method of characteristics boundary calculation in a finite difference calculation method has deterred almost all researchers. An approach is derived here which requires little modification to present programs and allows the use of the exact boundary condition calculation. The analysis is based on the recognition that the characteristic equations and variables are imbedded in the finite difference equations and can be extracted at very little expense. The same finite difference algorithm that is used at ordinary interior points is used to compute new values at the boundary point. These values do not, in general, match the boundary conditions, however, the velocity components and pressure can be combined into the correct value for the characteristic variables at that point. The appropriate boundary condition can then be applied. The utility and accuracy of the three dimensional characteristic boundary conditions are demonstrated on a variety of problems including flow inside a rectangular duct, three dimensional plane impingement flows, axisymmetric plume flow. For two dimensional cases there is excellent agreement with exact solutions. Comparisons of three dimensional cases with available exact solutions will be presented in the full paper.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Progress on the research effort "Analytical Investigation of Multinozzle Plume Flow Fields" is summarized. Major achievements include (a) the qualitative understanding of the complex shock structure impingement of two underexpanded rocket plumes and (b) the development of three dimensional computational tech- niques. A three dimensional isentropic inviscid supersonic computer code is operational and the core flow field of rectangular nozzles has been computed. Advanced techniques built around floating discrete discontinuity surfaces and singularity cells are being incorporated into this program for the computation of the impingement of two rectangular plumes.		

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